Environmental Bonds and the Incentive to Research in Activities Involving Uncertain Future Effects

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Introduction

It is intuitive that the difficulty of predicting the future outcomes of present activities will be greater the fewer the historical precedents for those activities. Activities for which there exist no historical precedents have been referred to by Shackle (1955, 1961) as 'crucial'. Because there exist no observations of the historical outcomes of Shackle-crucial activities, there is no basis on which to identify their possible outcomes or to construct a probability distribution for those outcomes. The only source of information on the future effects of such activities is the experimental research undertaken in advance of their introduction.

The problem addressed in this paper arises from the fact that in the absence of non-market incentives there is no reason to believe that experimental research conducted by the agents proposing such innovative activities will include all relevant potential future costs. Because the activities are historically unique there is no basis on which to establish *ex ante* markets in all potential future effects, hence there may be a range of unexpected social costs or benefits. Since the weakness of such markets open us the possibility of 'Thalidomide-type' surprises, it is worh considering whether there exists an incentive to research that will ensure that all socially relevant questions are asked about the future external effects of activities with no or few historical precedents. A passive learning process of the type discussed by Opaluch (1984) will certainly add information on the effects of innovative activities as those effects emerge, but it will not uncover in advance the possibility of future social costs, unless there exist incentives to research those costs.

The paper considers the use of sequentially determined environmental bonds as incentives to research the socially interesting outcomes of innovative activities. Environmental bonds of one sort or another have long been used to encourage socially desirable methods of waste disposal in activities where the existing waste disposal technologies have a range of social effects, some more harmful than others. Environmental bonds have not previously been considered as research incentives, but it turns out that they are very well suited to the purpose. The social insurance aspect of the bond is retained, but it is augmented by an uncertainty premium relating to the maximum conjectured future social costs of the activity, and this generates a direct incentive to experimental research.

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The argument is developed in two stages. The next section identifies a measure of the present social value of future external effects of activities with and without historical precedents. This covers certain issues previously addressed by Dasgupta and Heal (1979) and Bockstael and Opaluch (1983). The measure arrived at is a composite. All activities have at least some precedents - implying that bits of them have been done before. Hence the measure for all cases rests on an expected present social value of those future effects known to occur with some probability. On to this is grafted a measure of the conjectured present social value of the remaining (uncertain) outcomes. Section 3 considers how the measure fits the properties of environmental bonds developed out of the materials-use fee first recommended by Mill (1972) and Solow (1971). A final section offers some concluding remarks.

The present social value of future external effects

Let the technology of the ith economic activity in the period k be described by the time-indexed n-dimensional row vectors of input and ouput coefficients, $a_i(k)$ and $b_i(k)$. Assume that m components of these vectors, m < n. refer to economic (positively valued) inputs and outputs, and that the remaining n-m components refer to non-economic (zero-valued) inputs and outputs. The latter describe inputs and outputs for which there exist no well-defined private property rights - for whatever reason. We can thus identify a corresponding vector of prices, p(k), m components of which are positive, all others being equal to zero. The unit activity level price equation for the ith activity in period k is as follows:

$$[b_{i|P1} +, ..., + b_{imPm} + b_{im+1}^{0} +, ..., + b_{1n}^{0}](k) = (1+r)[a_{i|P1} +, ..., + a_{im}p_m + a_{im+10} +, ..., + a_{in}0](k) \qquad \dots (1)$$

in which r is the rate of profit on the m economic goods used in the activity.

Since the list of economic inputs and outputs is incomplete for m < n. there are up to n - m inputs and outputs which are unobserved through the price system. If these non-economic inputs or outputs are linked backwards or forwards to other economic activities we have the familiar problem of external effects. In other words, if the inputs allocated by the vector $[a_{im+1},...,a_{in}](k)$ affect the outputs allocated bu the vector $[b_{i1},...,b_{im}](k)$, and if those inputs are themselves the indirect product of some other economic activity, then the value of output of the ith activity may be said to depend on that other activity.

Following Meade's early (1952) contribution to the subject I assume that indirect external effects of this sort act to scale the output obtainable from a given set of economic inputs in the downstream activities. If the external effect is positive, output is scaled upwards. If it is negative, output is scaled downwards. Because an indirect link implies the existence of some intermediate activity, it also implies some delay in the transmission of the effects. Hence I take the scalar to be a function of the level of earlier economic activities. Accordingly, the vector of economic outputs of an activity may be described as a function of the economic inputs to that activity, scaled by the external effects of economic activities undertaken over some proceding interval. Formally, defining a time-indexed m-dimensional row vector of economic activity levels y(k), the choice variables of the economic system, we have

$$b_{ii} = f[a_i](k)g[y(k-t,k)], \ j = 1,...,m. \qquad ...(2)$$

What this says is that output of the jth economic good in the ith activity in period k is a function both of the combination of economic inputs in that period, and the external effects of economic activities undertaken over the previous k-t periods.

The length of the interval (k-t,k) will depend on the length of the chain connecting the activities of the economy. For any pair of economic activities it is assumed that there is a well-defined time-distance between them. The marginal output effects on $b_{ij}(k)$ of the level of activity in the jth process in period k-t will be given by the partial derivative $\partial g/\partial y_j(k-t)$. If less than zero it implies a negative external effect, if greater than zero it implies a positive external effect - where a negative external effect means that a marginal increase in activity levels in the economy reduces the economically valued output obtainable from a given set of economically valued inputs, while a positive external effect implies the opposite. For purposes of this paper I assume that uncertainly relates to the sign and magnitude of the derivatives of the function g[y(k-t,k)], and not to the time-distance between activities. The assumption that the timedistance between the emittor and recipient(s) of pollution if known with certainty implies an understanding of the existing structure of environmental systems, but not their propensity to mutate as a result of changes in the environmental inputs due to economic activities. Food chains, for example, are assumed to be reasonably well understood. But it is also assumed that we may not know what pollution of a system at some point along the food chain will mean for systems higher up the chain. More particularly, the future external effects of present activities will be uncertain wherever those activities are innovative.

The economy-environment model on which this description of external effects is based is reported in Perrings [1986.1987]. The principal implication of this model is that a system comprising an economy which is open to its environment will be characterised by persistent change in the conditions of production. The disposal of residuals generated in either the economy or its environment will force the general system to mutate. In the present case it is assumed that changes in the non-economic inputs to an economic activity simply scale the economic outputs of that activity. But there is no reason why the relative quantity of economic outputs should not change too. Similarly, there is no reason why the non-economic outputs of the process should remain the same either absolutely or relatively. The existence of future external effects is, in fact, merely a symptom of the evolutionary nature of the system.

A discussion of the motors of change in the system is beyond the scope of this paper, but it is worth noting that they rest on the necessity for residuals to be returned to the system causing change in both the level and input-output characteristics of all activities. The first is a scale effect, the second may be called an adaptation effect. Since it is only if the global system is fully controlled or if the economic system is isolated from its environment that residuals generated in the economy may be accommodated through change in the level of activity alone, and since the economy-environment system is neither controllable nor decomposable, change will typically involve both effects.

Consider, now, the construction of measures for the future external effects of present activities. Let us first take only those activities or aspects of activities for which there exist some historical precedents. If the time-distance between the jth and ith activities is t periods, a unit increase in the level of activity in the jth process in period k may be assumed (on the basis or historical experience) to generate a set of s possible outcomes in the ith process in period k+t. That is, we can identify a set of s vales for the output vector $\mathbf{b}_i(\mathbf{k+t})$ dependent on the level of activity $\mathbf{y}_i(\mathbf{k})$. We may denote this set of values $\mathbf{b}_{i,j}^{h}(\mathbf{k+t})$, $\mathbf{h} = 1,...,s$. in which the subscript j stands for $\mathbf{y}_i(\mathbf{k})$. It is subject to the probability distribution $\pi_{i,j} = \pi[\mathbf{b}_{i,j}(\mathbf{k+t})]$, with $\pi_{i,j}^{h} \ge 0$, and $\sum_{h} \pi_{i,j}^{h} = 1$. We thus have complete (historically acquired) knowledge of the set of outcomes possible for activity i in period k+t as a result of the level of activity in j in period k. From this it is possible to define the expected output losses or gains associated with that level of activity. We denote these expected output losses or gains

$$Eb_{i,i}^{*(k+t)} = Eb_{i,j=0}^{(k+t)} - Eb_{i,j=Yj(k)}^{(k+t)} \qquad \dots (3)$$

where $Eb_{i,j=0}(k+t)$ is expected output in the ith process when activity in the jth process is equal to zero, and $Eb_{i,j=Y(k)}^{(k+1)}$ is expected output in the ith process when activity in the jth process is at the actual level obtaining in period k.

In order to establish the value to society of the environmental external effects of such an activity, we need measures of both the welfare cost of the risks involved, $x_{i,j}(k)$, and the social rate of time preference, d. Derivation of the first of these measures for the particular case considered here is discussed in the appendix to this paper. The second is well enough understood to require no comment. Letting the weighting factor for the risk attached to expected future environmental external effects be a function, of the welfare cost to society of bearing that risk, the (undiscounted) expected social value of the effect at time k is the expected value of the effect weighted for the welfare costs of risk, $Ez_{i,j}(k+t)$. This is defined by

$$Ez_{i}(k+t) = Ey_{i}(k+t)[Eb_{i}^{*}(k+t)F(x_{i}(k))]Ep(k+t+1) \qquad \dots (4)$$

where $E_{y_i}(k+t)$ and $E_{p_i}(k+t+1)$ are the expected values of $y_i(k+t)$ and $p_i(k+t+1)$ respectively.

To obtain a present social value for the general future external effects of the jth activity at time k, we need only to discount this as the social rate of time preference and to sum over all affected downstream activities. Thus the expected present social value of the external effects of the jth activity in period k+t is given by

$$EZ_{i}(k+t) = \sum_{i} (Ey_{i}(k+t)[Eb_{i}]^{*}(k+t)F(x_{i}(k))]Ep(k+t+1))[1+d]^{k+t} \quad i = 1,...,m \quad ...(5)$$

This is all familiar territory and creates no particular difficulties. The problem is not so tractable, however, once we come to those activities for which the information set is far less complete than has been assumed so far. Recall that activities are defined to be Shackle-crucial if they are historically unique. Such activities are subject to uncertainty in the sense of Knight: neither the range nor the distribution (whether subjective or objective) of the outcomes of such activities is known. Decision makers are thus assumed to operate in ignorance of the outcomes. It is well known that there exists no algorithm for decision-making in such circumstances. The various algorithms canvassed in the literature on uncertainty all impose sufficient information to make decisions possible. The assignment of subjective probabilities, for example, requires individual intuitive assessment of the likelihood of any one of a number of (mutually exclusive) outcomes to eventuate. It requires, in other words, complete intuitive knowledge of the set of outcomes. Similarly, the assignment of equal probabilities to each of a number of conjectured outcomes on the assumption that there exists insufficient information to discrininate between them also implies a remarkably complete knowledge of the set of possible outcomes. The Arrow-Hurwicz [1972] approach to the problem of decision-making under ignorance makes perhaps the weakest knowledge assumptions, requiring only knowledge of the ordering between a number of pairs of outcomes. But it still necessitates what is a well defined set of options, each with a well defined range of possible outcomes.

The problem in wholly Shackle-crucial activities is precisely to generate sufficient information to support decision-making. That is the raison-d'etre for experimental research to support innovative activities. Since information does not exist is has to be acquired, and in innovative activities this implies advance experimental research. The economic problem is to evaluate the present value to society of such research. The solution suggested in this paper seeks a measure of this in the maximum conjectured costs to society of proceeding without such research.

In the face of complete ignorance it is reasonable to begin the process of acquiring socially relevant information by imagining the best (if society is risk avid) and worst (if society is risk averse) possible future outcomes from a given activity. To do this we may assign maximum or minimum conjectured values to the effects on the ith process in period k+t as a result of activities undertaken in the jth process in period k, $C_{max}b_{i,j}(k+t)$ or $C_{min}b_{i,j}(k+t)$. We have already assumed that there exists an expected output potential for the ith process in period k+t when $y_j(k) = 0$, $Eb_{i,i=0}(k+t)$. There are thus four cases of interest:

(i) if $C_{max}b_{i,j}(k+t) = C_{min}b_{i,j}(k+t) = Eb_{i,j=0}(k+t)$, activity j will be conjectured to have no effect on activity i;

(ii) if $Eb_{i,j}(k+t) > C_{max}b_{i,j}(k+t) \ge C_{min}b_{i,j}(k+t)$, activity j will be conjectured to have negative effects on activity i;

(iii) if $C_{\max} b_{i,j}(k+t) \ge C_{\min} b_{i,j}(k+t) > Eb_{i,j=0}(k+t)$, activity j will be conjectured to have positive effects on activity i; and

(iv) if $C_{\max}b_{i,j}(k+t) > Eb_{i,j=0}(k+t) > C_{\min}b_{i,j}(k+t)$, activity j will be conjectured to have effects that may be either positive or negative.

In each of the last three cases there will be potential gain in improving the quality of the information required to make a decision.

By similar construction to (5), and assuming a strictly concave (risk averse) social welfare function, the conjectured present social value of the external effects of the jth activity occuring in t periods is

$$CZ_{i}(k+t) = \sum_{i} (Ey_{i}(k+t)[Eb_{i}]_{i=0}(k+t) - C_{min}b_{i}(k+t)]Ep(k+t+1)[1+d]^{k+t} \quad i = 1,...,m. \quad ...(6)$$

This gives the discounted social vale of the maximum conjectured loss associated with the unresearched implementation of the jth activity. In cases (ii) and (iv) the term $[Eb_{i,j=0}(k+t) - C_{min}b_{i,j}(k+t)]$ is positive, in case (iii) it is negative.

Thus for activities with both Shackle-crucial and Shackle-non crucial elements the appropriate measure is rather ungainly but straightforward

$$\begin{split} & EZ_{j}(k+t) + CZ_{j}(k+t) = \sum_{i} (Ey_{i}(k+t) \{ (Eb_{i,j}^{*}(k+t)F(x_{i,j}(k))) \} + [Eb_{i,j}(k+t) - C_{min}b_{i,j}(k+t))] Ep(k+t+1)[1+d]^{-k+t} \dots (7) \\ & i = 1, \dots, m \end{split}$$

This is different from the sum of (5) and (6) only in that in that the term $Eb_{i,j=0}(k+t)$ in (6), the expected output of the ith activity in period k+t at zero levels of activity in the jth process at period k, is replaced by $Eb_{i,j}(k+t)$ in (7), the expected ouput of the ith activity in period k+t net of the expected effects of positive levels of activity in j. This because the maximum conjectured loss refers to the residual uncertainty only.

This last measure will be the relevant one wherever there do not exist sufficient observations to estimate an expected value for the future costs of present activities within acceptable limits of confidence. In all such cases the expected present social value of the future effects of current activities should be augmented by the maximum conjectured losses associated with the Shackle-crucial aspects of those activities.

Environmental bonds and the incentive to research

Following the insights in the work of Boulding [1966] and Ayres and Kneese [1969] as to the significance of the law of conservation of mass for the waste disposal problem, Mills [1972] and Solow [1971] separately advanced the idea of a 'materials-use fee' to be levied on specified environmental resources at a rate equal to 'the social cost to the environment if the material were eventually returned to the environment in the most harmful way possible' [Solow, 1971, 502]. The fee was initially seen to be equivalent to the refundable deposit long used to encourage the recycling of potentially environmental resources to dispose of waste products in a socially preferred way. The fee was prompted by considerations similar to those behind the use of Pigouvian taxes for pollution control including, in particular, the non-existence of an enforceable contractual obligation on the users of a resource to perform in predictable way. Where the purchaser of a resource was contractually free to dispose of it in any of a number of ways, each with different known effects, the materials-use fee provided an incentive to adopt the least socially harmful method of disposal. The fee, now commonly referred to as an environmental bond, has subsequently been recommended wherever direct observation and detection of environmental damage are impossible or extremely difficult [cf Baumol and Oates 1975, 1979].

To establish the basis for the particular bonds recommended below it is useful to remind ourselves that whatever the form of the levy or subsidy attached to an expected environmental external effect, it may be interpreted as a premium for social insurance against the losses due to acts of commission or omission. Losses due to acts of commission are the consequence of negative external effects, losses due to acts of omission are the positive external effects forgone if a programme of activity generating positive effects is not undertaken. In the case of non-innovative activities the data set may be assumed to be sufficiently rich that the expected value of an external effect is known, and the premium may be accurately computed. In the case of innovative activities this is not the case.

In innovative activities the problem is to ensure sufficient advance experimental research to avert unexpected future losses to society. This requires the generation of a sufficiently rich data set to enable the estimation of an expected present value for the social costs of such activities within acceptable confidence limits. The particular form of the environmental bond that would seem to be appropriate in these circumstances is one that varies with the maximum conjectured loss associated with the Shacklecrucial aspects of activities. Since the maximum conjectured loss depends on the information available to the environmental authority, such a bond would also vary with that information. To ensure that new information was incorporated in such a bond, it would have to be regularly reviewed. What seems to be called for, therefore, is a sequentially determined environmental bond that reflects the maximum conjectured loss to society of allowing an activity to proceed without further inquiry. Such a bond would establish a measure of the social importance attached to research on the outcomes of an activity that would vary with the state of knowledge. The motivation for this is quite simple. Where the maximum conjectured losses caused by any activity with potential future environmental effects are substantial, care should be taken in constructing a reliable data set on which to found decisions. Maximum conjectured losses that are insignificant do not warrant extensive initial investigation. Maximum conjectured losses that are catastrophic demand a major research effort.

The research incentive in such a bond derives from the fact that it would vary with the information available to the environmental authority. The use of maximum conjectured losses as the basis for calculating bonds in innovative activities accordingly represents no more than a place to start. Since the value of a bond would be sensitive to research which reduces the residual uncertainly attached to an activity, a substantial initial bond might be revised downwards if experimental research eliminated any of that residual uncertainty.

From the previous section if the jth activity has both innovative and non-innovative elements with effects t periods in the future, the value of the bond on those effects at time k will be a function of two things: the expected losses due to the non-innovative parts of the activity, and the maximum conjectured losses due to the innovative parts of the activity. Denoting the bond by $W_i(k+t)$, this implies that

$$W_{i}(k+t) = G[EZ_{i}(k+t)+CZ_{i}(k+t)].$$
 ...(8)

G' > 0, G(0) = 0. More generally, since the value of the bond on the effects of activity in the jth process occurring with a delay of t periods is reassessed in each period, it may be revised up to t times. It may accordingly be described by the sequence

$$(W_{i}(k+t))^{t} = G((Z_{i}(k+t)+CZ_{i}(k+t))^{t}), \quad t = 0...t.$$
 ...(9)

Because the future value of the bond may vary, the agents operating the process will be influenced in their decision-making by their own expectations of those future values, or more particularly by their own expectations of the future cost of the bond. This cost comprises both the opportunity cost of the current bond, the expected present value of the opportunity cost and refund losses of the bond in future periods, and the present value of research expenditures over the life of the bond. The opportunity cost of the bond in any one period is the difference between the interest it could earn if invested at the market rate and the interest actually paid by the environmental authority. If we assume that interest is paid by the environmental authority at the social discount rate, d, and if this is less than the market rate, r, this cost in period $k+\tau$ is simply $W_{k+\tau}(r-d)$. The agents' own expectation of the value of the bond in the same period may be denoted E W ($k+\tau$), and their expectation of the amount surrendered to the authority at the refund date is E W (k+t). The latter is a function of the subjective probability assigned by the agents undertaking the activity to the maximum loss actually occurring. Research expenditure in period k+t is denoted $R_{1}(k+\tau)$. The incentive effects of the bonds depend on the relation between these different elements in the cost of the bond. Research that lowers the value of the bond by reducing the residual uncertainty attached to an activity may have both direct effects on the opportunity cost and the expected refund losses of the bond, but it also has indirect effects on profits via its implications for the level of activity. Since, from (2), the external effects of any activity are assumed to increase with the level of that activity, firms will maximize profits through choice of the level of activity. If experimental research causes the environmental authority to reduce the value of the bond, it will also enable the firm to operate at a higher level of activity. The net result of a change in research expenditure depends on a number of contradictory effects. In all cases, however, research expenditure will be increased up to the point at which the benefits from research (due to the increase in profits associated with the increase in the level of activity plus the reduction in the expected value of the refund loss) are equal to the costs due to direct research expenditures and the change in the opportunity of cost of the bond (caused by a change in the level of activity in the jth process, and by the provision of additional information to the environmental authority).

Aside from the level of activity the vale of the bond in future periods would depend on two things. First, in respect of the Shackle-non crucial aspects of the activity there would be a passive learning process of the type analysed by Opaluch [1984]. Second, because the bond includes a premium for residual uncertainty, research which reduces uncertainty would reduce that premium. Symmetrically, because the bond is designed to indemnify society against effects which cannot be specified contractually, and reduction of uncertainty enables the substitution of direct contractual arrangements for the indirect incentives of the bond. The net effect is that bonds of this type will be sequentially determined, reflecting both changes in the level of activity of the offending processes and the level of residual uncertainty associated with those processes. Indeed, the only circumstance in which the bond is unlikely to change over the course

of a programme of activity is where the time-distance between the affected activities is significantly greater than the expected life of the project. In such cases there may be no possibility of learning more about the effect within the expected life of the project, and it may be reasonable to impose a bond of fixed value.

Concluding remarks

It is widely held that the external effects of information, together with problems of appropriability and moral hazard, ensure that competitive markets will lead private agents to avoid investment in basic research, and to overspecialize in applied research [cf Dasgupta and Heal, 1979]. This creates particular problems in innovative activities with environmental effects that are conjectured to occur with some delay. The delays are significant for a number of reasons. The most important is that they heighten uncertainly and so encourage a myopic vision of the future. The more myopic the vision of competitive agents, the less the incentive to undertake basic experimental research. Moreover, the more that private rates of time preference are driven above the social rate of time preference, the lower the probability that internalisation of external effects by the assignment of private property wights will be socially optimal [cf Perrings, 1987; Seneca and Taussig, 1984; Fisher, 1981].

Since there is no reason to believe that private agents will invest in experimental research at socially optimal levels in cases where the external effects of current activities may be significantly delayed, it is worth considering whether there exist incentives to ensure that due weight is given to the social importance of research in innovative activities. This paper constructs a measure of the present social value of the future expected and potential external effects of non-innovative and innovative activities, respectively, and treats this as a proxy for the social value of research. It is suggested that this measure form the basis for the sequential calculation of environmental bonds.

Given the motivation of the paper, two properties of such environmental bonds turn out to be of particular interest. First, by implicitly weighting the 'worst case' or the maximum conjectured loss associated with any activity at unity, the bonds enable a risk adverse society to signal to private agents the social value placed on advance experimental research in cases where the outcome is uncertain. Second, given that the expected value of the future external effects of activities is contingent on the set of current relative prices facing the agents undertaking those activities, the bonds enable the environmental authority to change the distribution of outcomes by changing current relative prices. These properties would seem to make the bonds useful both in preventing innovators from evading the potential costs of activities undertaken in ignorance, and of avoiding the worst of those potential costs.

The measures for the bonds derived in this paper imply an initially very cautious approach to innovative activities. Since the value of the bonds is sensitive to the quantity and quality of information generated in advance research programmes, however, only inadequately researched innovative activities would be penalised. The warrant for such a conservative approach comes from the risk aversion implicit in the assumption of a strictly concave social welfare function. If society were risk avid (implying a strictly convex social welfare function) it would be appropriate to focus on the maximum conjectured gains, not losses, associated with the future external effects of current activities. There are obviously enough examples of risk avid societies to make the latter case credible, though the enormous welfare costs that have historically resulted from risk avid behaviour should make one wary of promoting it. Finally, it is worth noting that the irreducible uncertainty of innovative activities in an evolutionary system means that there will always exist the possibility of surprise. The environmental bonds recommended in this paper would not eliminate this possibility, but they would provide the incentive to firms to anticipate so far as possible the future outcomes of present activities, regardless of the time-horizon employed in their own planning process.

Appendix

To establish a premium for risk aversion from the social welfare function of the collectivity, assume a function of the form

$$\mathbf{V}(\mathbf{k}) = \sum_{i} \pi_{i} \mathbf{V}[\mathbf{y}_{i}(\mathbf{k})]. \tag{1}$$

Welfare is an increasing, strictly concave, continously differentiable function of y(k). If $Eb_{i,j}(k+t)$ is the expected value of the output in the ith process in period k+t as a result of external effects deriving from activities in the jth process in period k. then because of the strict concavity of V(k).

$$V[y_{i}(k+t)Eb_{i,j}(k+t)] > \sum_{h} \pi_{i}^{h} V[y_{i}(k+t)b_{i,j}^{h}(k+t)], h = 1,..,s.$$
(2)

There will therefore exist a positive number $x_{i,i}(k)$ such that

$$V[y_{i}(k+t)Eb_{i,i}(k+t)] - x_{i,i}(k)] = \sum_{h} \pi_{i}^{h} V[y_{i}(k+t)b_{i,i}^{h}(k+t)], \ h = 1,..,s.$$
(3)

Writing (3) in the form

$$V[y_{i}(k+t)Eb_{i,j}(k+t)] - x_{i,j}(k)] - V[y_{i}(k+t)Eb_{i,j}(k+t)] = \sum_{h} \pi_{i}^{h}V[y_{i}(k+t)b_{i,j}^{h}(k+t)] - V[y_{i}(k+t)Eb_{i,j}(k+t)]$$

and expanding both sides in Taylor series implies, where $x_{i,j}(k)$ is small enough to ignore second and higher powers, that

$$-\mathbf{x}_{i,i}(\mathbf{k})\mathbf{V}'[\mathbf{y}_{i}(\mathbf{k}+t)\mathbf{E}\mathbf{b}_{i,i}(\mathbf{k}+t)] = 1/2 \ \mathbf{V}''[\mathbf{y}_{i}(\mathbf{k}+t)\mathbf{E}\mathbf{b}_{i,i}(\mathbf{k}+t)\mathbf{s}^{2}[\mathbf{b}_{i,i}(\mathbf{k}+t)] \qquad \dots (4)$$

 $s^{2}[b_{i,j}(k+t)]$ being the variance of output in the ith process in period k+t net of external effects deriving from activity in the jth process in period k.

Hence $x_{i,j}(k)$, the welfare cost of the risks in the ith process in period k+t associated with activity in the jth process in period k is defined by:

$$x_{i,j}(k) = -1/2 - \frac{V''[y_i(k+t)Eb_{i,j}(k+t)s^2[b_{i,j}(k+t)]]}{V'[y_i(k+t)Eb_{i,j}(k+t)]} \qquad ...(5)$$

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